MATHEMATICAL MODEL OF THERMOSTATS AND ITS NUMERICAL REALIZATION

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Principles for the construction of thermal and mathematical models and procedures of numerical calculation of thermostats are presented. The results of the calculations are compared with experimental data.

At present, the thermal conditions of a thermostat are calculated on the basis of approximate analytic methods which do not allow a three-dimensional temperature field of the thermostatted object to be obtained, as is required for optical crystals, for example. In addition, there is not uncommonly also a need to consider three-dimensional temperature fields for other components of the thermostat (for example, the thermostat chamber), since the distribution of these fields may affect the quality of operation of the thermostated object.

Further development of the theory of the thermal conditions of thermostats may be achieved on the basis of approximate numerical methods, which, as is known, are broader in their range of application than analytic methods. In this context, four basic problems may be distinguished:

1) establishing thermal and mathematical models of thermostats oriented toward the use of numerical methods, and applying these models in an automatic-projection system;
2) developing procedures for numerical calculation of the mathematical models;
3) choosing the structure of the library of programs realizing the numerical procedures;
4) creating a library of programs which will allow the thermal conditions of a broad class of thermostats to be calculated.

The thermostats most widely used at present are heating, liquid, and semiconductor thermostats, and those using gas-filled thermal tubes. Often a single thermostat will use several means of heat intake and extraction. Usually in a thermostat, a conventional scheme of which is shown in Fig. 1, a series of basic elements may be distinguished [1]: the thermostatted object (optical crystal, electronic circuit, quartz resonator, etc.); the thermostat chamber, in which the thermostatted object is placed; the shell surrounding the chamber, which may take the form of heat-insulating shells; the chamber of the second thermostatting circuit, etc.; the element performing heat intake or extraction (thermopile, heater, etc.), automatic regulator, heat detector. Real thermostats are widely different both in the combination of component elements and in the structure of the thermal connections between them. This variety, and also the difficulty of determining the thermal conditions of the thermostat by numerical means, impose constraints on the means of solving the above problems. Approaches in which, for each specific thermostat, a new calculation program completely unrelated to the programs for other thermostats must be written are unsuitable. A more rational approach is to synthesize the calculation program for the thermal conditions of a specific thermostat on the basis of a library of modules. This library would contain modules for the most frequently encountered elements and connections. Provision must also be made for the inclusion in the library of additional modules corresponding to elements and thermal-connection structures which either are so far absent from the library or else are refinements of existing models. The same framework underlies the approach adopted in constructing thermal and mathematical models and developing procedures for numerical calculation.

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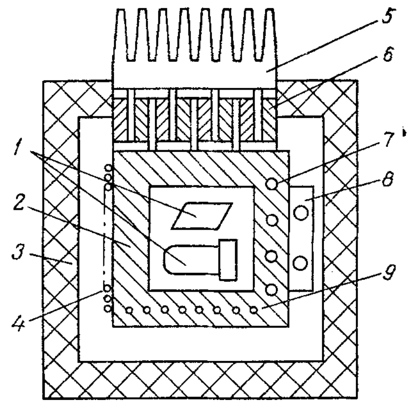

Fig. 1. Schematic model of thermostat: 1) thermostatted object; 2) chamber; 3) insulating shell; 4) surface heater; 5) radiator; 6) semiconducting thermoelectric thermopile; 7) channels for coolant; 8) thermal tube; 9) internal heater.

Thermal and Mathematical Model

In this approach it is expedient to represent the thermal model of a specific thermostat as a composite of the thermal models of the elements and the connections between them, and to construct it on the basis of these models. Hence, this gives rise to the problem of isolating the thermal elements and connections and developing the corresponding models for them. In doing so, it is necessary to impose a number of constraints which must be satisfied by the models of typical elements and connections in order to ensure that they are compatible for use in synthesizing the model of a specific thermostat. Taking into account the features of real numerical methods and the specific features of thermostats, the following constraints are adopted:

a) in all the component parts of thermostat elements made from solid materials, heat propagation is by conduction;

b) the absorption of radiation by any element of a thermostat occurs on its surface;

c) gaps between thermostat elements are filled with materials of negligibly small specific heat;

d) in formulating the boundary conditions, the heat flux density for any point on the surface of a component due to heat exchange with other components may be regarded as depending on the temperature of this point and the temperature fields on the surface of the other components.

The mathematical model of a specific thermostat is constructed from equations for the typical elements and thermal connections between them appearing in its thermal model. As an example, consider the model of a typical component, a rectangular shell (which might be the thermostat chamber, the heat-insulating shells, etc.), and a model of the typical thermal connection between rectangular cells.

A rectangular shell will be understood to be a shell in which the inner and outer surfaces are the surfaces of rectangular parallelepipeds. The presence of three rectangular holes is permitted in the shell, but no more than one hole may be on any one face, two are on opposite faces, and their surfaces are parallel to the corresponding faces of the shell. The shell walls may be made of different materials. Thermal contact between the parts of a single shell is assumed to be ideal. In writing the mathematical model of the surface heater, consideration is given to both surface heat sources and, when the heater is placed inside the shell, volume heat sources. For a liquid thermostat, the action of the liquid coolant passing along channels inside the shell is assumed to be equivalent to the action of a volume heat sink (the model of a volume heat sink was considered in [2]).

The equations for the shell temperature field take the form

$$c(\bar{x}, T)\rho(\bar{x})\frac{\partial T}{\partial \tau} = \sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(\lambda(\bar{x}, T)\frac{\partial T}{\partial x_j}\right) + q_v(\bar{x}, T, T_1, \ldots, T_n), \tag{1}$$

$$-\lambda(\bar{x})\frac{\partial T}{\partial n} = \alpha(\bar{x}, T, T_1, \ldots, T_n)(T - t(\bar{x}, T_1, \ldots, T_n)) + q_s(\bar{x}, T_1, \ldots, T_n), \tag{2}$$

$$\bar{x} = (x_1, x_2, x_3),$$

where q_V, α, t, and q_S are functions reflecting the effect of interacting components on the shell. With the appropriate choice of functions, the action of a heater placed inside the shell or a coolant passing through the channels in the shell may be described; using q_S, the action of a heater on the shell or the heat flux at the thermostat body may be described, and using the heat-transfer coefficient α and the temperature t the heat-transfer of the shell with the surrounding elements.

To obtain a thermal model of the interaction between two rectangular shells, it is assumed that the thickness of the gap between them may be neglected, the heat fluxes for all forms of heat transfer is normal to the surface, and the values of the local heat-transfer coefficients are determined solely by the temperatures of points of the interacting shells lying on a common normal to their surfaces.

With these simplifications, the boundary condition describing the action of neighboring shells on the shell being considered is of the form

$$-\lambda(\bar{x}, T)\frac{\partial T}{\partial n} = \alpha(\bar{x}, T, T^{\mathrm{nei}})(T - T^{\mathrm{nei}}), \tag{3}$$

where T^{nei} is the temperature of a point of the surface of the neighboring shell lying on a normal to the surface of the shell being considered, that passes through the point x.

Procedure for Numerical Calculation

The mathematical model of a thermostat in considering three-dimensional temperature fields consists basically of equations of parabolic type, which typically have the form in Eqs. (1) and (2). In the literature, sufficient attention has been given to methods of constructing difference schemes for systems of equations of parabolic type in the case when certain unknown functions appear in the boundary conditions of equations of other functions.

The main difficulty in constructing difference schemes for Eqs. (1) and (2) is the need to approximate the functions α, t, q_V, and q_S, which depend on the temperature fields of the other bodies. Of the available possibilities for such an approximation, the simplest for program realization is that involving the use of grid functions corresponding to the temperature fields of other bodies for an earlier instant of time. Then the distributions of α, t, qv, and q_S must be regarded as unknowns in determining the temperature field of the component at the given moment of time, i.e., the problem in Eqs. (1) and (2) actually reduces to the following

$$c(\bar{x}, T)\rho(\bar{x}, T)\frac{\partial T}{\partial \tau} = \sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(\lambda(\bar{x}, T)\frac{\partial T}{\partial x_j}\right) + q_v'(\bar{x}, \tau), \tag{4}$$

$$-\lambda(\bar{x}, T)\frac{\partial T}{\partial n} = \alpha'(x, \tau)(T - t'(\bar{x}, \tau)) + q_s'(\bar{x}, \tau), \tag{5}$$

where α', t', q_V', and q_S' are known functions of the spatial coordinates and the time. This opens the possibility of developing a universal program for calculating the temperature field of the component, which may be used without change in calculating the thermal conditions of any specific thermostat. To secure universality, it is sufficient to use an appropriate numerical method for the solution of Eqs. (4) and (5) and to specify, in writing the program, that α', t', q_V', and q_S' may be arbitrary functions of the spatial coordinates and time. In using such a program to calculate a specific thermostat, grid analogs of the functions α', t', q_V', and q_S' are formed at each time step in terms of the temperature fields of the other elements, in accordance with the characteristic thermal connections.

Algorithms for the formation of these analogs, which are called conforming algorithms, are determined by the functions α, t, q_V, and q_S from Eqs. (1) and (2) which appear in the

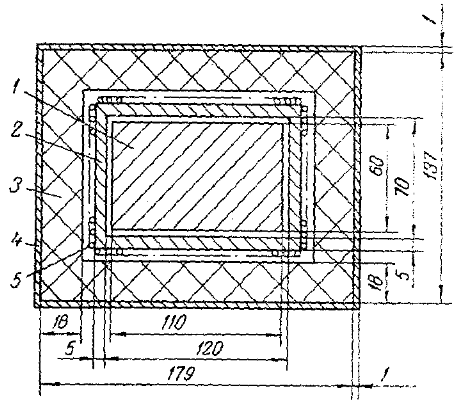

Fig. 2. Thermal model of thermostat: 1) thermostatted object; 2) chamber; 3) insulating shell; 4) body of thermostat; 5) heater.

boundary conditions of the equations for the elements and the distribution of volume heat sources (or sinks). In the conforming algorithms, some standardization may also be introduced, reducing them to typical form. In the majority of cases encountered in practice, the functions describing the thermal connections between components are additive

$$\alpha(\bar{x}, T, T_1, \ldots, T_n) = \sum_{i=1}^{n} \alpha_i(\bar{x}, T, T_i), \quad q_v(\bar{x}, T, T_1, \ldots, T_n) =$$
$$= \sum_{i=1}^{n} q_v(\bar{x}, T, T_i), \quad q_s(\bar{x}, T_1, T_2, \ldots, T_n) = \sum_{i=1}^{n} q_{si}(\bar{x}, T_i). \tag{6}$$

Thus, any function reflecting the action of other bodies on the given component comprises a sum, each term of which depends only on the temperature of a point of the body and the temperature field of the bodies acting. Conforming algorithms may also be additive, i.e., may resolve into a series of independent algorithms, each of which corresponds to the contribution from the action of the given component with one of the bodies which act upon it. Correspondingly, conforming programs are additive. By this means, it is possible to develop conforming algorithms and programs reflecting more extensive thermal connections [for rectangular shells, the typical connection is described by Eq. (3)], and to assemble conforming programs from these typical programs common to all thermostats.

Structure of Library

In using the above numerical procedure, the program for calculating the thermal conditions of a specific thermostat includes calculation programs and conforming programs. Correspondingly, the library will consist of modules of two types, for calculating typical elements and for matching the elements for typical thermal connections between them. Such a library may be said to have a block structure, since the program for the calculation of a specific thermostat is assembled from ready-made modules — blocks. The structure of the library allows it to be fairly easily filled with calculation programs for diverse thermostatting elements, heat sensors, thermostatted objects, etc., and programs corresponding to typical forms of thermal connections between the thermostat components.

Results of Investigation

To illustrate the machine time required for the analysis of thermostat temperature fields by the given method, a calculation has been carried out for the thermostat with the thermal model shown in Fig. 2. The thermostatting element 1 is in the form of a rectangular parallelepiped and is surrounded by three shells of rectangular form (thermostat chamber 2, insulating layer 3, thermostat body 4). On the external surface of chamber 2, a uniformly distributed heater of power 6 W is fitted. The thermal conductivities of bodies 1-4 are as follows: 170, 170, 0.06, and 45 W/mK; the total specific heats are 928, 588, 409, and 504 J/°K; the heat-transfer coefficients between bodies 1 and 2, and 2 and 3 are 8 $W/m^2 \cdot {}^\circ K$ and that between the body and the surrounding medium is 4 $W/m^2 \cdot {}^\circ K$. Thermal contact between the body and the insulation is assumed to be ideal. Initially, the temperature of all the elements is the temperature of the surrounding medium.

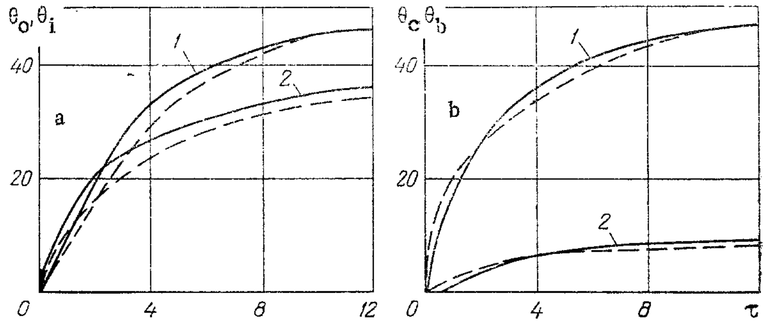

Fig. 3. Dependence of the temperature differences θ_o, θ_i (a) and θ_c, θ_b (b) with respect to the medium on the time τ: curve 1 shows θ_o (a) and θ_c (b), while curve 2 shows θ_i (a) and θ_b (b); the continuous curves are the result of calculation and the dashed curves of experiment: θ_o, θ_i, θ_b, θ_c, °K; τ, sec.

The thermal model described above is a composite of the thermal model of elements of rectangular-shell type and of an element of the type of the thermostatted object in the form of a rectangular parallelepiped, and thermal models of the interactions between them. The thermal model of an element of the type of the thermostatted object in the form of a rectangular parallelepiped and the thermal model of the interaction between the object and the shell are analogous to the corresponding models for shells described above.

It follows from the structure of the thermostat thermal model that the calculation program is constructed of four module types: for the calculation of an element of rectangular-shell type, for the calculation of an element of the type of the thermostatted object in the form of a rectangular parallelepiped, and for matching the thermal connections between the rectangular shells and the thermostatted object. The calculation program for the thermostat was assembled from these modules of the library.

The change in temperature fields of the elements after switching on a heater before the system reached steady conditions was investigated experimentally and by subsequent calculation.

Curves of the difference between the mean surface temperatures of the elements and the temperature of the external medium are given in Fig. 3; the calculations involved approximately 1 h of machine time on an ES-1022 computer.

It is evident from these curves that the agreement between the calculated and experimental data is satisfactory for engineering purposes. This, and the amount of machine time required, lead to the conclusion that the proposed method of investigating the three-dimensional temperature fields of thermostats is efficient.

NOTATION

T, T_1, ..., T_n, temperature fields of the shell and the elements interacting with it; c, λ, ρ, specific heat, heat conduction, and density of shell material; α, coefficient of convective heat transfer; q_v, q_s, specific volume and surface energy sources (sinks); x_1, x_2, x_3, spatial coordinates; τ, time; θ_c, θ_b, θ_o, θ_i, differences from the surrounding medium of the mean surface temperatures of the chamber, the thermostat body, the thermostatted object, and the insulating shell.

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